CS 188: Artificial Intelligence
Spring 2010

Lecture 24: Perceptrons and More!

4/22/2010

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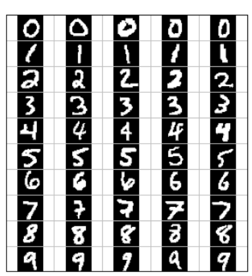
Slides adapted from Dan Klein

Announcements

- W7 due tonight [this is your last written for the semester!]
- Project 5 out tonight --- Classification!



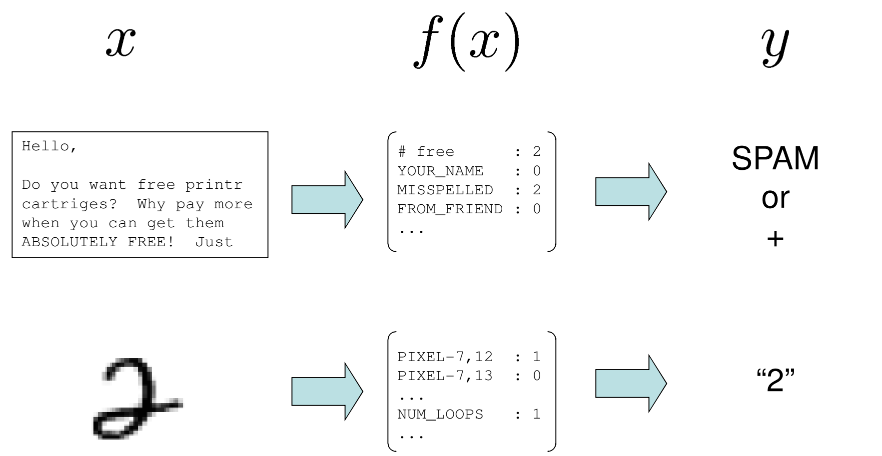
Announcements (2)

- Contest logistics
  - Up and running!
  - Tournaments every night
  - Final tournament: We will use submissions received by Thursday May 6, 11pm.
- Contest extra credit through bonus points on final exam [all based on final ranking]
  - 0.5pt for beating Staff
  - 0.5pt for beating Fa09-TeamA (top 5), Fa09-TeamB (top 10), and Fa09-TeamC (top 20) from last semester [total of 1.5pts to be earned]
  - 1pt for being 3^{rd}
  - 2pts for being 2^{nd}
  - 3pts for being 1^{st}

Where are we and what's left?

- So far:
  - Search
  - CSPs
  - Adversarial search
  - MDPs and RL
  - Bayes nets, probabilistic inference
  - Machine learning – classification
- Today: Machine Learning part III:
  - kNN and kernels
- Tuesday: Applications in Robotics
- Thursday: Applications in Vision and Language + Conclusion + Where to learn more

Classification

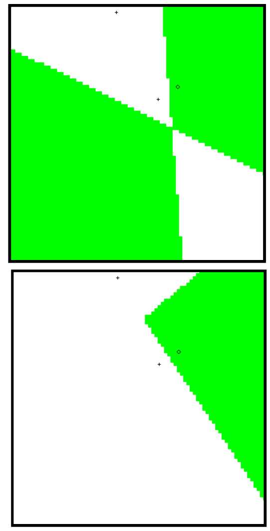


Classification overview

- Naïve Bayes:
  - Builds a model training data
  - Gives prediction probabilities
  - Strong assumptions about feature independence
  - One pass through data (counting)
- Perceptron:
  - Makes less assumptions about data
  - Mistake-driven learning
  - Multiple passes through data (prediction)
  - Often more accurate
- SVM:
  - Properties similar to perceptron
  - Convex optimization formulation
- Nearest-Neighbor:
  - Non-parametric: more expressive with more training data
- Kernels
  - Efficient way to make linear learning architectures into nonlinear ones

Case-Based Reasoning

- Similarity for classification
  - Case-based reasoning
  - Predict an instance's label using similar instances
- Nearest-neighbor classification
  - 1-NN: copy the label of the most similar data point
  - K-NN: let the k nearest neighbors vote (have to devise a weighting scheme)
  - Key issue: how to define similarity
  - Trade-off:
    - Small k gives relevant neighbors
    - Large k gives smoother functions
    - Sound familiar?
- [Demo]

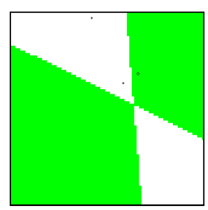


10

http://www.cs.cmu.edu/~zhuxj/courseproject/knndemo/KNN.html

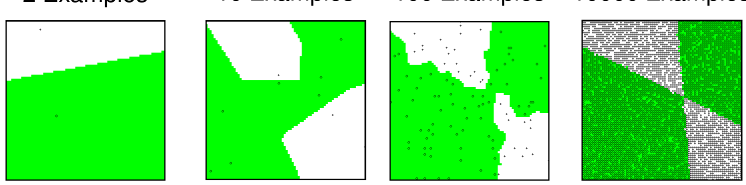
Parametric / Non-parametric

- Parametric models: prior
  - Fixed set of parameters
  - More data means better settings
- Non-parametric models:
  - Complexity of the classifier increases with data
  - Better in the limit, often worse in the non-limit
- (K)NN is non-parametric



Truth

2 Examples | 10 Examples | 100 Examples | 10000 Examples



11

Nearest-Neighbor Classification

- Nearest neighbor for digits:
  - Take new image
  - Compare to all training images
  - Assign based on closest example
- Encoding: image is vector of intensities:

 $= \langle 0.0\ 0.0\ 0.3\ 0.8\ 0.7\ 0.1 \ldots 0.0 \rangle$

- What's the similarity function?
  - Dot product of two images vectors?

$$\text{sim}(x, x') = x \cdot x' = \sum_i x_i x'_i$$

  - Usually normalize vectors so ||x|| = 1
  - min = 0 (when?), max = 1 (when?)

12

Basic Similarity

- Many similarities based on feature dot products:

$$\text{sim}(x, x') = f(x) \cdot f(x') = \sum_i f_i(x) f_i(x')$$

- If features are just the pixels:

$$\text{sim}(x, x') = x \cdot x' = \sum_i x_i x'_i$$

- Note: not all similarities are of this form

13

Invariant Metrics

- Better distances use knowledge about vision
- Invariant metrics:
  - Similarities are invariant under certain transformations
  - Rotation, scaling, translation, stroke-thickness…
  - E.g:  
    - 16 x 16 = 256 pixels; a point in 256-dim space
    - Small similarity in R^{256} (why?)
  - Variety of invariant metrics in literature
- Viable alternative: transform training examples such that training set includes all variations

14

Classification overview

- Naïve Bayes
- Perceptron
- SVM
- Nearest-Neighbor
- ***Kernels***

[learning similarity metrics
massive scaling]

A Tale of Two Approaches …

- Nearest neighbor-like approaches
  - Can use fancy similarity functions
  - Don't actually get to do explicit learning
- Perceptron-like approaches
  - Explicit training to reduce empirical error
  - Can't use fancy similarity, only linear
  - Or can they? Let's find out!

19

Perceptron Weights

- What is the final value of a weight w_y of a perceptron?
  - Can it be any real vector?
  - No! It's built by adding up inputs.

$$w_y = 0 + \underbrace{f(x_1)} - \underbrace{f(x_5)} + \ldots$$

$\alpha_{1,y} = 1$
$\alpha_{2,y} = 0$
$\alpha_{3,y} = 0$
$\alpha_{4,y} = 0$
$\alpha_{5,y} = -1$
...

$$w_y = \sum_i \alpha_{i,y} f(x_i)$$

- Can reconstruct weight vectors (the primal representation) from update counts (the dual representation)

$$\alpha_y = \langle \alpha_{1,y} \;\; \alpha_{2,y} \;\; \ldots \;\; \alpha_{n,y} \rangle$$

20

Dual Perceptron

$(a+b)\cdot c = a\cdot c + b\cdot c$

- How to classify a new example x?

$$
\begin{aligned}
\text{score}(y,x) &= w_y \cdot f(x) \\
&= \left(\sum_i \alpha_{i,y}\, f(x_i)\right) \cdot f(x) \\
&= \sum_i \alpha_{i,y}\, \underbrace{(f(x_i)\cdot f(x))}_{\text{inner product}} \quad \text{Kernel function} \\
&= \sum_i \alpha_{i,y}\, K(x_i, x)
\end{aligned}
$$

- If someone tells us the value of K for each pair of examples, never need to build the weight vectors!

21

Dual Perceptron

- Start with zero counts (alpha) $\alpha_{y_1}, \alpha_{y_2}, \ldots, \alpha_{y_k}$
- Pick up training instances one by one
- Try to classify x_n,

$$y = \arg\max_y \sum_i \alpha_{i,y}\, K(x_i, x_n) \qquad f(x_i)\cdot f(x_n)$$

- If correct, no change!
- If wrong: lower count of wrong class (for this instance), raise score of right class (for this instance)

$$\alpha_{y,n} = \alpha_{y,n} - 1 \qquad w_y = w_y - f(x_n)$$

$$\alpha_{y^*,n} = \alpha_{y^*,n} + 1 \qquad w_{y^*} = w_{y^*} + f(x_n)$$

22

$\|x_i - x\|_2^2 = (x_i - x)^T (x_i - x) = x_i^T x_i - 2 x_i^T x + x^T x = K(x_i, x_i) - 2K(x_i, x) + K(x, x)$

Kernelized Perceptron

- If we had a black box (kernel) which told us the dot product of two examples x and y:
  - Could work entirely with the dual representation
  - No need to ever take dot products ("kernel trick")

$$\text{score}(y, x) = w_y \cdot f(x)$$

$$= \sum_i \alpha_{i,y} \; K(x_i, x)$$

- Like nearest neighbor – work with black-box similarities
- Downside: slow if many examples get nonzero alpha

23

$K(x_i, x_j) = f(x_i) \cdot f(x_j)$

Kernels: Who Cares?

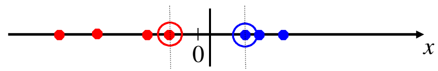
- So far: a very strange way of doing a very simple calculation
- "Kernel trick": we can substitute any* similarity function in place of the dot product
- Lets us learn new kinds of hypothesis

* Fine print: if your kernel doesn't satisfy certain technical requirements, lots of proofs break. E.g. convergence, mistake bounds. In practice, illegal kernels *sometimes* work (but not always).

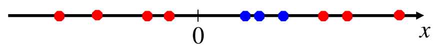
25

Non-Linear Separators

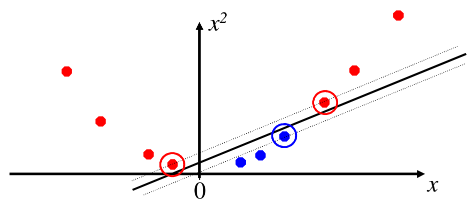
- Data that is linearly separable (with some noise) works out great:



- But what are we going to do if the dataset is just too hard?



- How about… mapping data to a higher-dimensional space:

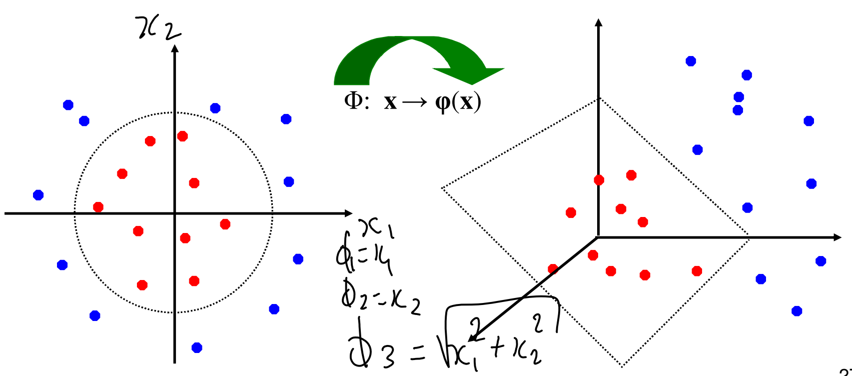


26

This and next few slides adapted from Ray Mooney, UT

Non-Linear Separators

- General idea: the original feature space can always be mapped to some higher-dimensional feature space where the training set is separable:



27

Some Kernels

- Kernels implicitly map original vectors to higher dimensional spaces, take the dot product there, and hand the result back

- Linear kernel: $K(x, x') = x \cdot x' = \sum_i x_i x'_i$

$$\rightarrow \quad \phi(x) = x$$

- Quadratic kernel: $K(x, x') = (x \cdot x' + 1)^2$

$$= \sum_{i,j} \underbrace{x_i x_j}\, \underbrace{x'_i x'_j} + 2 \sum_i \underbrace{x_i}\, \underbrace{x'_i} + \underbrace{1}$$

For $x \in \Re^3$:

$$\phi(x) = [x_1 x_1 \; x_1 x_2 \; x_1 x_3 \; x_2 x_1 \; x_2 x_2 \; x_2 x_3 \; x_3 x_1 \; x_3 x_2 \; x_3 x_3 \; \sqrt{2}x_1 \; \sqrt{2}x_2 \; \sqrt{2}x_3 \; 1]$$

$\in \mathbb{R}^{13}$

28

Some Kernels (2)

- Polynomial kernel: $K(x, x') = \underbrace{(x \cdot x' + 1)^d}_{\phi(x) \cdot \phi(x')}$

For $x \in \Re^3$:

$$\rightarrow \quad \phi(x) = [x_1^d \; x_2^d \; x_3^d \; \sqrt{d}x_1^{d-1}x_2 \; \sqrt{d}x_1^{d-1}x_3 \; \ldots \; \sqrt{d}x_1 \; \sqrt{d}x_2 \; \sqrt{d}x_3 \; 1]$$

$n = 3$

$d = 1000$ $\quad \binom{1003}{1000}$

For $x \in \Re^n$ the d-order polynomial kernel's implicit feature space is $\binom{n+d}{d}$ dimensional.

By contrast, computing the kernel directly only requires $O(n)$ time.

Some Kernels (3)

- Kernels implicitly map original vectors to higher dimensional spaces, take the dot product there, and hand the result back
- Radial Basis Function (or Gaussian) Kernel: infinite dimensional representation

$$\longrightarrow \; K(x, x') = \exp(-||x - x'||^2)$$

- Discrete kernels: e.g. string kernels
  - Features: all possible strings up to some length
  - To compute kernel: don't need to enumerate all substrings for each word, but only need to find strings appearing in both x and x'

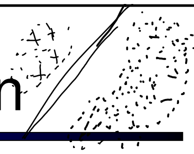
30

Why Kernels?

- Can't you just add these features on your own (e.g. add all pairs of features instead of using the quadratic kernel)?
  - Yes, in principle, just compute them
  - No need to modify any algorithms
  - But, number of features can get large (or infinite)
- Kernels let us compute with these features implicitly
  - Example: implicit dot product in polynomial, Gaussian and string kernel takes much less space and time per dot product
  - Of course, there's the cost for using the pure dual algorithms: you need to compute the similarity to every training datum

"$K(x, x')$ is a Mercer kernel iff $\exists\, \phi$: $K(x,x') = \phi(x) \cdot \phi(x')$ $\forall x, x'$"

Recap: Classification

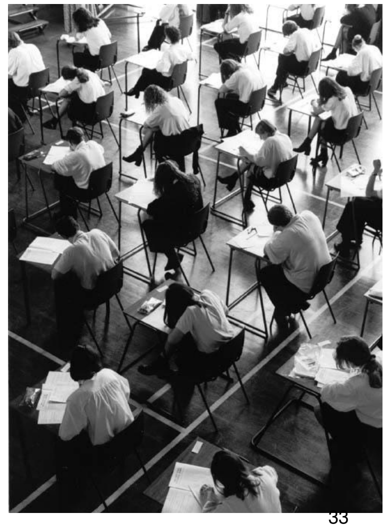


- Classification systems:
  - Supervised learning
  - Make a prediction given evidence
  - We've seen several methods for this
  - Useful when you have labeled data

Outside of scope of 188:

unsupervised learning $\{x_i\}_{i=1}^n$

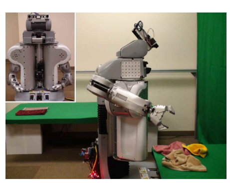
semi-supervised learning $\{(x_i, y_i)\}_{i=1}^m$ $\{x_j\}_{j=1}^n$



33

Where are we and what's left?

- So far foundations: Search, CSPs, Adversarial search, MDPs and RL, Bayes nets and probabilistic inference, Machine learning
- Tuesday: Applications in Robotics



- Thursday: Applications in Vision and Language + Conclusion + Where/How to learn more